

MATH 3/4: ASSIGNMENT 14

MAY 2, 2010

PROPERTIES OF OPERATIONS

In mathematics and other sciences we often use letters instead of numbers to show that some kind of relationship will work for all numbers or some group of numbers. These letters are called *variables*. A mathematical expression that shows relationship between variables is called a formula.

Recall, for example, formulas connecting subtraction and addition, and formulas connecting division and multiplication.

If a number c is a sum of addends a and b , then

$$a + b = c$$

$$c - a = b$$

$$c - b = a$$

If a number c is a product of factors a and b , then

$$a \times b = c$$

$$c \div a = b$$

$$c \div b = a$$

Now let us talk about properties of each operation separately.

PROPERTIES OF ADDITION

Commutative property: changing the order of addends does not change the sum.

Formula: $a + c = c + a$

Example: $2 + 4 = 4 + 2 = 6$

Associative property: changing the grouping of addends does not change the sum.

Formula: $(a + b) + c = a + (b + c)$

Example: $(3 + 6) + 5 = 9 + 5 = 14$; $3 + (6 + 5) = 3 + 11 = 14$; $(3 + 6) + 5 = 3 + (6 + 5)$

PROPERTIES OF MULTIPLICATION

Commutative property: changing the order of factors does not change the product.

Formula: $a \times c = c \times a$

Example: $2 \times 4 = 4 \times 2 = 8$; $3 \times 7 = 7 \times 3 = 21$

Associative property: changing the grouping of factors does not change the product.

Formula: $(a \times b) \times c = a \times (b \times c)$

Example: $(3 \times 6) \times 5 = 18 \times 5 = 90$; $3 \times (6 \times 5) = 3 \times 30 = 90$; $(3 \times 6) \times 5 = 3 \times (6 \times 5)$

Distributive property of multiplication over addition: sum of two numbers times a third number equals product of the first and third numbers plus product of the second and third numbers.

Formula: $(a + b) \times c = a \times c + b \times c$

Example: $(2 + 3) \times 5 = 5 \times 5 = 25$; $2 \times 5 + 3 \times 5 = 10 + 15 = 25 \rightarrow (2 + 3) \times 5 = 2 \times 5 + 3 \times 5$

Distributive property of multiplication over subtraction: difference of two numbers times a third number equals product of the first and third numbers minus product of the second and third numbers.

Formula: $(a - b) \times c = a \times c - b \times c$

Example: $(6 - 3) \times 5 = 3 \times 5 = 15$; $6 \times 5 - 3 \times 5 = 30 - 15 = 15 \rightarrow (6 - 3) \times 5 = 6 \times 5 - 3 \times 5$

OPERATIONS WITH NUMBERS 0 AND 1

Multiplication or division by 1 does not change the number: For every number a , $a \times 1 = a$ and $a \div 1 = a$

Example: $7 \times 1 = 7$ and $7 \div 1 = 7$

Adding or subtracting 0 does not change the number: For every number a , $a + 0 = a$ and $a - 0 = a$

Example: $7 + 0 = 7$ and $7 - 0 = 7$

Multiplying or dividing 0 by another number gives you 0: For every number a , $0 \times a = 0$ and $0 \div a = 0$. Since multiplication is commutative, for every number a , $a \times 0 = 0$.

Example: $0 \times 7 = 7 \times 0 = 0$ and $0 \div 7 = 0$

Dividing by 0 is not defined in mathematics: It means, $7 \div 0$ does not equal anything. There is a reason for that. Let us assume that there is such number b that $7 \div 0 = b$. Then the equation $b \times 0 = 7$ must be true. But we know, that $b \times 0 = 0$ for any b . So we got a contradiction.

1. Fill in the blanks:

- (a) $(9 + 7) \times 2 = \underline{\hspace{2cm}} \times 2 = \underline{\hspace{2cm}}$
- (b) $9 \times 2 + 7 \times 2 = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
- (c) $(4 + 6) \times 4 = \underline{\hspace{2cm}} * 4 = \underline{\hspace{2cm}}$
- (d) $4 \times 4 + 6 \times 4 = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

2. In the airport a passenger weighed his baggage, consisting of a bag, a suitcase and a backpack. The weight of all three pieces was 20 kg. The bag weights 4 kg 200g and the suitcase is twice as heavy as the bag. For this problem:

- (a) explain in words, what numerical expressions $4200 * 2$ and $20\ 000 - 4200$ mean;
- (b) explain in words, what numerical expression $4200 * 2 + 4200$ means;
- (c) write a numerical expression that represents how heavy the backpack is.

3. The weight of a barrel of cider is 25kg 100g. The weight of the empty barrel is 9 kg 140g. One liter of cider weights 798g. Write a numerical expression that represents how many liters of cider are in the barrel. Find the value of the numerical expression.

4. There was 5 tons 200 kg of wheat in two barns. Farmers Bob and Billy took some wheat from each barn. They divided this wheat into two equal parts and loaded two trucks equally. After that there were 2750 kg of wheat left in one barn and 550 kg in the other barn. Write a numerical expression that represents how many kg of wheat was loaded on each truck. Find the value of the numerical expression.

5. Michael has four more nickels than quarters. This amounts to \$1.40. Find the number of quarters and the number of nickels.

6. A gentleman gave 6 cents each to his little grandchildren; had he given them 10 cents each, it would have taken 28 cents more; how many little grandchildren does he have?

7. "Give me two dollars", said a boy to his sister. Then we'll have equal amounts of money! The sister gave him two dollars. How much more money did the sister have in the beginning?

8. Sylvia has exactly \$1.00 in change. She has at least one penny, one nickel, one dime, and one quarter. She does not have any half dollars. What is the least number of coins she could have?

9. A boy and a girl were saving up money to buy a game. The boy was \$7 short and the girl was \$2 short and together they still did not have enough money to buy a game. What was the cost of the game? The price is the whole number of dollars.

10. The Candy game. Two players are playing in Candy game. They bite in turn from a candy - natural number. Each player can bite no more than a half of candy left. The one who cannot bite and leave a piece of candy for the opponent loses.

- (a) Play Candy" several time starting from 15, 20 and 25
- (b) Which positions are losing for this type of Candy? What should the strategy of the player be to win?