

MATH 3/4: ASSIGNMENT 18

JUNE 6, 2010

BEGINNING PROBABILITY THEORY

We will be talking about “tests” (such as tossing a coin, rolling a die, drawing a card, etc), each of which can result in one of several possible outcomes (e.g., rolling a die can give numbers 1 through 6). If there are n possible outcomes, and they are all equally likely, then probability of getting any given one is exactly $1/n$; for example, probability of rolling a 3 on a die is $1/6$.

In most cases, we will be interested in probability of something that can be obtained in more than one way. For example, if we ask what is the probability of rolling an even number on a die, then there are 3 ways to get it: by rolling 2, 4, or 6. Each of these outcomes has probability $1/6$, so the probability of getting one of them is $1/6 + 1/6 + 1/6 = 3/6 = 1/2$.

Question: we roll a die twice. What is the probability of getting 2 on the first roll and 3 on the second?

Solution: rolling a die twice gives us a pair of numbers, each from 1 to 6. We will write the pairs like this: (2, 3). We need to compute how many such pairs are there. The easiest way is to arrange them in a table like this:

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	...			(2, 6)
(3, 1)	(3, 2)	...			(3, 6)
(4, 1)	(4, 2)	...			(4, 6)
(5, 1)	(5, 2)	...			(5, 6)
(6, 1)	(6, 2)	...			(6, 6)

There are 6 rows and 6 columns, so there are $6 \times 6 = 36$ possible pairs. Therefore, the probability of getting any one of them (e.g., (2, 3)) is $\frac{1}{36}$.

Question: we roll two dice. What is the probability of rolling a 5 and a 6?

Answer: let us label the dice, for example labeling one of them as die number 1, the other as die number 2. Then there are two ways of getting a 5 and a 6: as pair (5, 6) (5 on die number 1, 6 on die number 2) or as (6, 5) (6 on die number 1, 5 on die number 2). Thus, the answer is $\frac{2}{36}$.

Here is one more example.

Question: we roll two dice. What is the probability of getting sum of two numbers equal to 4?

Answer: there 3 ways of getting sum 4: (1, 3), (2, 2), (3, 1). Thus the probability is $\frac{3}{36} = \frac{1}{12}$.

Homework problems on back

HOMEWORK

- In the game of roulette, there are 37 slots, numbered 0 through 36. Of numbers 1–36, half are red, the other half are black (zero has no color). What is the probability of hitting
 - A number between 1–12
 - An even number other than zero
 - A red number or zero
 - If you bet \$15 on odd numbers (i.e., you win if you roll one of odd numbers), what is the probability of losing?
- You roll two dice, one red, one black. What is the probability of rolling two ones? Of rolling a 4 and a 6?
- The standard card deck has 4 suits (hearts, diamonds, spades, and clubs); each suit has 13 different card values: 2 through 10, jack, queen, king, and ace.

If you randomly draw one card, what is the probability of getting

 - The queen of spades
 - A face card (i.e., jack, queen, or king)
 - A black king
 - Anything but the queen of hearts
- I had drawn a card from the deck, and it turned out to be an ace. Now I am drawing one more card from the same deck. What is the probability that it will be an ace again?
- What is the probability that a randomly chosen person was born
 - in January?
 - on Feb 5?
 - on Sunday?

When doing this problem, you can ignore leap years and assume that birthdays are randomly distributed among all days of the year, so each day is equally likely; in real life it is not quite true.
- When rolling the standard die, what is the probability of rolling
 - an even number?
 - a number divisible by 3?
 - a number which is divisible by 2 or 3?
- If we roll two dice, what is the probability that the product of two numbers is a multiple of 3?
- If we roll two dice, what is the probability of getting sum equal to 11? equal to 9? equal to 7?